Negative Absorption Due to Coulomb Scattering of an Electron Stream

TOSHIMITSU MUSHA AND FUJIO YOSHIDA
Electrical Communication Laboratory, Musashino-shi, Tokyo, Japan
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Radio-frequency electrical conductivities are calculated with electrons drifting through an ion cloud under an externally applied static electric field. The stationary velocity distribution of electrons is taken to be a displaced Maxwellian. Then, if the drift velocity is larger than 0.8 times the electron velocity spread, the real part of the electrical conductivity becomes negative at frequencies much higher than the mean-collision frequency.

1. INTRODUCTION

NEGATIVE absorption of electromagnetic waves by an ionized gas with and without the presence of a magnetic field was first discussed by Twiss[1] and the possibility of such absorption by a partially ionized gas with the Ramsauer effect was suggested by Bekefi, Hirshfield, and Brown.[2] According to one of their results, the radiation temperature may be negative at frequencies below a certain value near the mean-collision frequency between the electrons and the gas atoms. On the other hand, the negative radiation temperature can be related to the negative electrical conductivity. The possibility of negative absorption in a fully ionized plasma in the absence of a static magnetic field was denied by Twiss[1,3] and Bekefi *et al.*[2] for an isotropic velocity distribution of electrons. Browne[4,5] derived the opposite result, but, (judging from later discussions by Marcuse,[6] and Mallozzi and Margenau[7]), it seems that his conclusion is not correct. Marcuse calculated in the Born approximation the possibilities for induced emission and absorption by a single-electron passing ions and concluded that the difference between the cross sections for emission and absorption becomes negative if the electric field of the incident electromagnetic wave is nearly parallel to the electron orbit. He showed further that electrons distributed isotropically in the velocity space always absorb part of the energy of the electromagnetic wave. In his calculation the frequency limits for negative absorption were not deduced and the velocity distribution of electrons was not taken into account.

All the work mentioned above is concerned only with the isotropic velocity distribution of electrons in a plasma. Sometimes electrons in a fully ionized plasma in the presence of a static electric field drift at a velocity which is comparable with or larger than the velocity spread.[8,9] In such cases the real part of the electrical conductivity of the plasma for high-frequency electromagnetic fields may become negative. Hence, the incident electromagnetic waves are amplified on passing through such a plasma. In this paper the authors will show this possibility for a fully ionized gaseous plasma and for a semiconductor plasma.

2. ELECTRICAL CONDUCTIVITY AND NEGATIVE ABSORPTION

Part of the energy of electromagnetic waves propagating through a passive medium is always absorbed by the medium. The absorbed power which is transformed into heat Q per unit time and volume is expressed as[10]

$$Q=(\omega/8\pi)(\epsilon''|E|^2+\mu''|H|^2)\,, \tag{1}$$

where ω is the angular frequency of the wave, ϵ'' and μ'' the imaginary parts of the electric and magnetic permeabilities, respectively, $|E|$ and $|H|$ the amplitudes of the electric and magnetic fields, respectively. Usually μ'' is much smaller than ϵ'' so that the second term in the bracket of Eq. (1) can be neglected. Since ϵ'' is related to the high-frequency electrical conductivity σ as

$$\epsilon''=(4\pi/\omega)\sigma \tag{2}$$

Q may be rewritten

$$Q=\tfrac{1}{2}\sigma|E|^2. \tag{3}$$

Consequently, if the conductivity becomes negative, the power flows from the medium into the field at a rate proportional to the energy density of the field. This results in amplification of the wave and is called "negative absorption." It is clear that a medium in a thermodynamic equilibrium can not exhibit negative conductivity.

Let us consider a stationary electron stream flowing through a gas along the x axis suffering collisions under a static electric field. This electron stream with a drift velocity v_d and a velocity spread v_{th} is shown in Fig. 1 in the velocity space. When an oscillating electric field E_1, which is in the direction of the x axis, is superimposed on the static electric field, the velocity distribution oscillates in the velocity space, giving rise to the polarization current. We examine qualitatively the relation

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[3] R. Q. Twiss, Astrophys. J. **136**, 438 (1962).
[2] G. Bekefi, J. L. Hirshfield, and S. C. Brown, Phys. Fluids **4**, 173 (1961).
[4] P. F. Browne, Astrophys. J. **134**, 963 (1961).
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[6] D. Marcuse, Bell System Tech. J. **41**, 1557 (1962).
[7] P. Mallozzi and H. Margenau, Astrophys. J. **137**, 851 (1963).
[8] H. Dreicer, Phys. Rev. **115**, 238 (1959).
[9] M. Glicksman and W. A. Hicinbothem, Phys. Rev. **129**, 1572 (1963).
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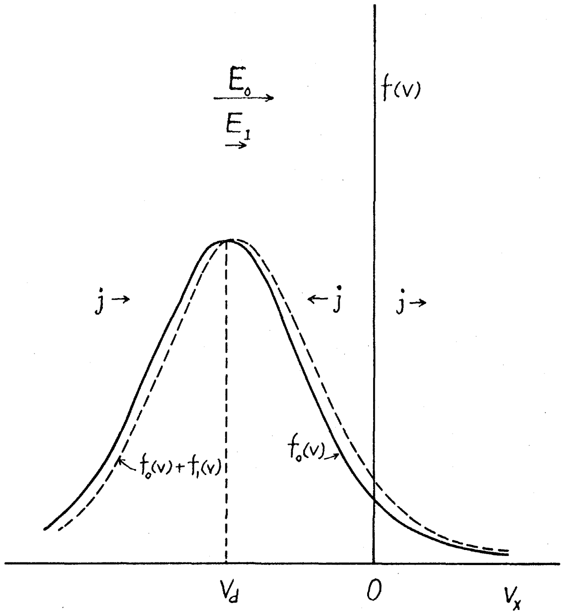

FIG. 1. The velocity distribution f_0 of drifting electrons under a static electric field E_0. When the fluctuating electric field E_1 is superposed upon E_0 the velocity distribution fluctuates in the velocity space. If this happens in phase with the electric force upon the electrons, the induced fluctuating electric current j caused by electrons with velocities between v_d and 0 flows in the opposite direction to E_1.

of the polarization current to the collision probability between the electrons and the gas atoms.

For frequencies extremely low compared with the mean-collision frequency of electrons, the velocity distribution fluctuates in phase with $-eE_1$. In this case the electrons with velocities between 0 and v_d cause a current component which is in the opposite direction to E_1 as is seen from Fig. 1. If the contribution of this component to the total current exceeds the resultant current due to all other parts of the velocity distribution, the current flows in the opposite direction to E_1 and negative conductivity results.[11] When the frequency of the electric field is much lower than the collision frequency, collisions decrease the amplitude of fluctuations of the velocity distribution, and consequently, the induced electric current. As a result, if the collision probability increases rapidly with the electron speed where the velocity distribution function of electrons has appreciable magnitudes, the negative contribution to the total current may overcome those from the other parts of the distribution. Negative conductivities in this sense have already been confirmed experimentally by Forman[12,13] and Ohara[14,15] by the use of electron streams through argon, krypton, and xenon gases with the mean-electron energies near the minima of collision probabilities of the respective gases.

At frequencies higher than the mean-collision frequency, the situation differs from that described above. At such frequencies the medium is, in general, inductive and collisions give rise to a component of the electric current in phase with the applied rf electric field. Accordingly, the role of collisions is reversed to that in the low-frequency case. Therefore, if the collision probability decreases rapidly with the electron speed where the velocity distribution function of electrons has appreciable magnitude, the negative contribution may overcome those from the other parts. The collision cross section by the Coulomb field is proportional to v^{-4} and it is expected that the electrical conductivity becomes negative if the drift velocity is large compared with the velocity spread. We calculate the electrical conductivity of electron streams flowing through a fully ionized plasma under a static electric field.

3. ELECTRICAL CONDUCTIVITY OF DRIFTING ELECTRONS

Let us consider a case where electrons are flowing through an ion cloud under a static electric field E_0 which is parallel to the x axis. Electrons are scattered by ions while accelerated by the external electric field and in an exact sense they will not reach a stationary state.[16] But, in practice, by virtue of scattering processes other than the Coulomb force they will become stationary. For simplicity the stationary velocity distribution of electrons is taken to be Maxwellian around the drift velocity v_d.

We begin by adopting the Boltzmann-Fokker-Planck equation to describe the electron dynamics, while the ions are regarded at rest and distributed randomly. There is, in addition to the static electric field, a prevailing uniform electric field oscillating in time at the frequency ω; that is, a transverse electromagnetic wave. The oscillating electric field E_1 is along the x axis. The calculation is confined to a small-amplitude electric field in the absence of an external magnetic field. Taking linear quantities with respect to fluctuating quantities in the usual Boltzmann-Fokker-Planck equation we find

$$\frac{\partial f_1}{\partial t}+(\mathbf{v}\cdot\nabla)f_1-\frac{e}{m}E_0\frac{\partial f_1}{\partial v_x}-\frac{e}{m}E_1\frac{\partial f_0}{\partial v_x}=\left(\frac{\partial f_1}{\partial t}\right)_{\text{coll}}. \quad (4)$$

Here f_1 is the small perturbation of the velocity distribution of the electrons by E_1, $-e$ and m are the electronic charge and mass, respectively, and f_0 is the stationary velocity distribution in the absence of E_1, given by

$$f_0=n_e(\beta/\pi)^{3/2}\exp[-\beta(\mathbf{v}-\mathbf{v}_d)^2] \quad (5)$$

$$\beta=m/2kT, \quad (6)$$

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[12] R. Forman, Phys. Rev. **128**, 1487 (1962).
[13] R. Forman, J. A. Ghormley, and J. R. Reiss, Phys. Rev. **128**, 1493 (1962).
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[16] H. Dreicer, Phys. Rev. **117**, 329 (1960).

where n_e is the electron density, k the Boltzmann constant, and T the transverse electron temperature.

We consider spatially uniform electric field or an electromagnetic wave, and in the latter case assume that the phase velocity is much larger than the electron velocities in the nonrelativistic plasma, so that the partial differentiation with respect to space coordinates could be neglected compared with the partial differentiation with respect to time. Then expressing the collision term on the right-hand side of Eq. (4) in a cylindrical coordinate system[17] we have, making use of the axial symmetry of the phenomenon

$$-i\omega f_1-\frac{e}{m}E_0\frac{1-\mu^2}{v}\frac{\partial f_1}{\partial\mu}-\frac{e}{m}E_0\mu\frac{\partial f_1}{\partial v}-\frac{e}{m}E_1\frac{1-\mu^2}{v}\frac{\partial f_0}{\partial\mu}-\frac{e}{m}E_1\mu\frac{\partial f_0}{\partial v}=n_i\Gamma\left\{\frac{1-\mu^2}{2v^3}\frac{\partial^2 f_1}{\partial\mu^2}-\frac{\mu}{v^3}\frac{\partial f_1}{\partial\mu}\right\} \quad (7)$$

$$\Gamma=4\pi e^4\ln\Lambda/m^2, \quad (8)$$

where Λ is the ratio of the maximum to minimum effective impact parameter. The maximum impact parameter is taken to be the Debye shielding length v_{th}/ω_p for frequencies smaller than the plasma frequency ω_p. For frequencies larger than the plasma frequency, however, it must be taken as v_{th}/ω,[18] so that

$$\Lambda=\begin{cases} mv_{th}^3/e^2\omega_p & \omega<\omega_p \\ mv_{th}^3/e^2\omega & \omega>\omega_p. \end{cases} \quad (9)$$

In Eq. (7) $\mu=\cos\theta$, where θ is the angle between the x axis and the direction of the electron velocity and n_i is the ion density. An arbitrary phase factor is involved in Eq. (7) except for $\exp(i\omega t)$ and this factor can be chosen so that E_1 is a real number except for $\exp(i\omega t)$. Let f_1 be divided into the real part f_1^r and the imaginary part f_1^i.

$$f_1=f_1^r+if_1^i. \quad (10)$$

Then f_1^r varies in phase with E_1 and f_1^i varies out of phase with E_1. Consequently Eq. (7) is decomposed into the real part

$$\omega f_1^i-\frac{e}{m}E_0\frac{1-\mu^2}{v}\frac{\partial f_1^r}{\partial\mu}-\frac{e}{m}E_0\mu\frac{\partial f_1^r}{\partial v}-\frac{e}{m}E_1\frac{1-\mu^2}{v}\frac{\partial f_0}{\partial\mu}-\frac{e}{m}E_1\mu\frac{\partial f_0}{\partial v}=n_i\Gamma\left\{\frac{1-\mu^2}{2v^3}\frac{\partial^2 f_1^r}{\partial\mu^2}-\frac{\mu}{v^3}\frac{\partial f_1^r}{\partial\mu}\right\}, \quad (11)$$

and the imaginary part

$$-\omega f_1^r-\frac{e}{m}E_0\frac{1-\mu^2}{v}\frac{\partial f_1^i}{\partial\mu}-\frac{e}{m}E_0\mu\frac{\partial f_1^i}{\partial v}=n_i\Gamma\left\{\frac{1-\mu^2}{2v^3}\frac{\partial^2 f_1^i}{\partial\mu^2}-\frac{\mu}{v^3}\frac{\partial f_1^i}{\partial\mu}\right\}. \quad (12)$$

We consider here the case when ω is much larger than the mean collision frequency. Then f_1^r is much smaller than f_1^i except for very slow electrons. In Eq. (11), neglecting terms which involve f_1^r compared with the other terms, we obtain the zeroth order $f_1^{i(0)}$. Next inserting $f_1^{i(0)}$ into Eq. (12) we have the first order $f_1^{r(1)}$. By iteration we can obtain the kth order quantities $f_1^{r(k)}$ and $f_1^{i(k)}$. Thus, the real part of the kth order fluctuation $f_1^{r(k)}$ implies three terms.

$$f_1^{r(k)}=f_{1,1}^{r(k-2)}+f_{1,2}^{r(k-2)}+f_{1,3}^{r(k-2)}+f_{1,1}^{r(k)}+f_{1,2}^{r(k)}+f_{1,3}^{r(k)} \quad (13)$$

for $k=3, 5, 7,\cdots$, together with

$$f_1^{r(1)}=f_{1,1}^{r(1)}+f_{1,3}^{r(1)}.$$

For $k\geqq 3$, these quantities are of the order

$$f_{1,1}^{r(k)}\sim\frac{1}{\omega^{k+1}}\left(\frac{e}{mv_{th}}\right)^{k+1}E_0^kE_1f_0 \quad (14)$$

$$f_{1,2}^{r(k)}\sim\frac{1}{\omega^{k+1}}\left(\frac{e}{mv_{th}}\right)^2E_0E_1\sum_{p,q,r}\left(\frac{n_i\Gamma}{v^3}\right)^p\left(\frac{n_i\Gamma}{v^2v_{th}}\right)^q\left(\frac{n_i\Gamma}{vv_{th}^2}\right)^r f_0$$
$$p+q+r=k-1;\ p,q,r=0,1,2,\cdots,k-1 \quad (15)$$

$$f_{1,3}^{r(k)}\sim\frac{1}{\omega^{k+1}}\left(\frac{eE_1}{mv_{th}}\right)\sum_{p,q,r}\left(\frac{n_i\Gamma}{v^3}\right)^p\left(\frac{n_i\Gamma}{v^2v_{th}}\right)^q\left(\frac{n_i\Gamma}{vv_{th}^2}\right)^r$$
$$p+q+r=k;\quad p=0,1,2,\cdots,k-1;\quad q,r=0,1,2,\cdots,k. \quad (16)$$

The first and the third terms represent effects of the static electric field and of collisions, respectively, on the energy exchange between the applied rf electric field and the ensemble of electrons. The second term expresses the combined effect of the electric field and the collisions.

If E_0 satisfies the following condition which is valid for an ordinary plasma

$$\omega\gg eE_0/mv_{th}, \quad (17)$$

$f_{1,1}^{r(k)}$ may well be approximated by $f_{1,1}^{r(1)}$. For an adequately large ω there is a critical electron speed v_c at which

$$\left.\begin{aligned} n_i\Gamma/v_c^3\omega\ll 1 \\ v_c<v_{th} \end{aligned}\right\}. \quad (18)$$

At such frequencies $f_{1,3}^{r(k)}$ may be approximated by $f_{1,3}^{r(1)}$, and $f_{1,2}^{r(k)}$ can be neglected compared with $f_{1,1}^{r(1)}$ for k equal to or larger than 3. Consequently

[17] M. N. Rosenbluth, W. M. MacDonald, and D. L. Judd, Phys. Rev. **107**, 1 (1957).
[18] D. B. Chang, Phys. Fluids **5**, 1558 (1962).

under both conditions Eqs. (17) and (18), $f_1{}^r$ can be approximated by $f_{1,1}{}^{r(1)}+f_{1,3}{}^{r(1)}$.

The electrical conductivity σ is defined with the velocity distribution of electrons as

$$\sigma=-\frac{e}{E_1}\int v_x f_1{}^r d\mathbf{v}. \quad (19)$$

Integration of $f_{1,1}{}^{r(1)}$ with respect to v is carried from 0 to infinity while integration of $f_{1,3}{}^{r(1)}$ with respect to v is carried from v_c to infinity. For $0<v<v_c$, $f_{1,3}{}^{r(1)}$ is not a good expression and the contribution from this part to the conductivity is considered later. Then the electrical conducitvity σ is calculated as (see Appendix)

$$\sigma=X(\omega,n_e,n_i,T)Y(v_d,v_c,T), \quad (20)$$

$$X=8\pi^2 n_e n_i e^6(\beta/\pi)^{3/2}\ln\Lambda/m^3\omega^2, \quad (21)$$

$$Y=(\xi\eta)^{-1}(1+\xi^{-2})\exp[-(\xi^2+\eta^2)]\sinh(2\xi\eta) -\xi^{-3}\int_{-\xi+\eta}^{\xi+\eta}\exp(-x^2)dx, \quad (22)$$

$$\xi=\beta^{1/2}v_d=1.224v_d/v_{th}$$
$$\eta=\beta^{1/2}v_c.$$

The behavior of Y versus v_d/v_{th} is shown in Fig. 2 for $v_c/v_{th}=\frac{1}{10}$, $\frac{1}{5}$, and $\frac{1}{3}$. It is clear that these curves do not largely depend upon the values of v_c/v_{th}. Figure 2 shows that the electrical conductivity becomes negative if the drift velocity is larger than 0.8 times the thermal velocity. This result is as expected in §2.

We refer here to the contribution to the electrical conductivity of electrons with speed between 0 and v_c which has been neglected in Eq. (20). For $-v_c\leqslant v_x\leqslant v_c$ in Fig. 1 any electrons at the right-hand side of $v_x=0$ have corresponding electrons at the left-hand side which have the same collision probabilities. Since df_0/dv is larger at the left-hand side, the electric current induced by E_1 in this velocity region is always in the opposite

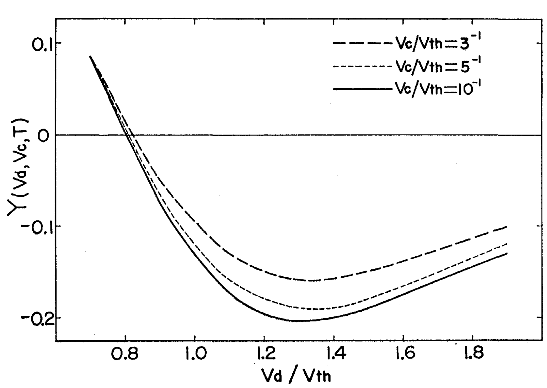

FIG. 2. Y of Eq. (22) versus v_d/v_{th}. When the drift velocity, v_d, becomes larger than 0.8 times the transverse thermal velocity, v_{th}, the electrical conductivity becomes negative.

direction to E_1. Therefore, the omitted integral in Eq. (20) would make the value of Y slightly smaller and the real cross point where $\sigma=0$ would be shifted slightly to the left of the calculated values in Fig. 2.

The values of X are plotted in Figs. 3 and 4 for different values of the electron temperature and of the electron density which is put equal to the ion density. Curves in these figure deviate from straight lines for $\omega>\omega_p$ since the value of Λ is given by Eq. (9). The lowest frequency above which the conductivity becomes negative is put to be 10 times the critical collision frequency $n_i\Gamma/v_c{}^3$ while the highest available frequency is limited by $\ln\Lambda$ since at too large frequencies $\ln\Lambda$ becomes so small that the expression of collision terms as Eq. (7) is not valid. We take the highest frequency so that $\ln\Lambda$ is about 5.

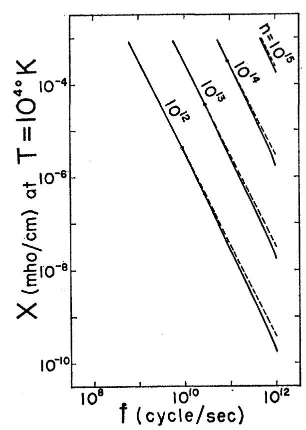

FIG. 3. X of Eq. (21) versus the frequency $\omega/2\pi$ at $T=10^4$°K. n is the electron density in cm^{-3} which is assumed to be equal to the ion density. Values of X deviate from straight lines at $\omega>\omega_p$ [see Eq. (9)].

4. DISCUSSION

When electrons and ions are flowing relative to one other, incident electromagnetic waves may be amplified if the relative drift velocity is larger than 0.8 times the velocity spread of the electrons. This amplification of waves should not be attributed to the two-stream instability but rather to collisions between electrons and ions. The space-charge effect has nothing to do with the negative conductivity. At ordinary experimental conditions in our laboratories, frequencies of incident electromagnetic waves which are to be amplified are in the microwave region as is seen in Figs. 3 and 4. The negative electrical conductivity can not only amplify the incident electromagnetic waves or externally applied rf electric fields, but also can excite oscillations if there are any oscillators inside or outside the plasma. Plasma oscillations may be excited by the negative conductivity since they are a kind of internal oscillator in the plasma. Amplification and excitation of electromagnetic waves propagating through a plasma with internal relative motion between electrons and ions may give rise to cosmic radiations.

Now let us consider a semiconductor plasma. Princi-

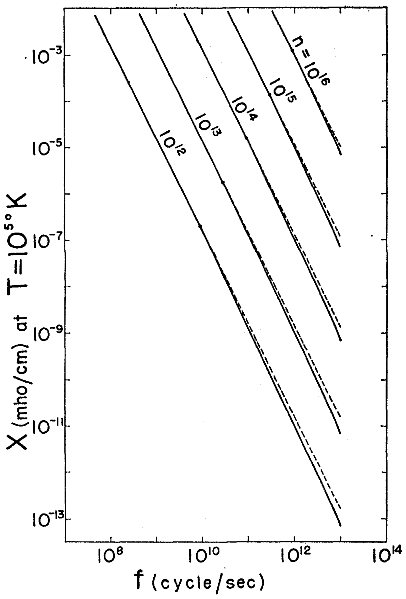

FIG. 4. X of Eq. (21) versus the frequency $\omega/2\pi$ at $T=10^5\,°\text{K}$.

pal scattering processes of electrons in semiconductors are lattice scattering by acoustic modes and by optical modes, impurity scattering, and carrier-carrier scattering.[19] At very low temperatures ionized-impurity scattering and electron-hole scattering become dominant. Recently Glicksman and Hicinbothem[9] reported that the drift velocity of electrons in indium antimonide becomes 1.5 times the transverse electron temperature. From this result the authors expect that indium antimonide at very low temperatures under a static electric field may have negative conductivity which could excite plasma oscillations in it at some tens kilomegacycles per second for impurity concentrations $10^{13}\sim10^{14}/\text{cm}^3$.

It seems very interesting to the authors that collisions do not always absorb part of the energy of the incident wave, i.e., randomize the coherent motion of electrons excited by the incident-wave field.

[19] C. Hilsum and A. C. Rose-Innes, *Semiconducting III-V Compounds* (Pergamon Press, Inc., New York, 1961).

ACKNOWLEDGMENTS

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APPENDIX

Since $f_{1,2}{}^{r(1)}$ vanishes, the electrical conductivity σ is calculated to the first order from Eq. (19) as

$$\sigma=-\frac{2\pi e}{E_1}\left\{\int_{-1}^{1}d\mu\int_0^\infty dv v^3\mu f_{1,1}{}^{r(1)}+\int_{-1}^{1}d\mu\int_{v_c}^\infty dv v^3\mu f_{1,3}{}^{r(1)}\right\}. \quad \text{(A1)}$$

From Eqs. (5), (11), and (12) $f_{1,1}{}^{r(1)}$ is given as

$$f_{1,1}{}^{r(1)}=-\frac{e}{m\omega}E_0\mu\frac{\partial f_1{}^{i(0)}}{\partial v}-\frac{e}{m\omega}E_0\frac{1-\mu^2}{v}\frac{\partial f_1{}^{i(0)}}{\partial\mu}$$
$$=-\left(\frac{e}{m\omega}\right)^2E_0E_1\{4\beta^2\mu^2v^2+8\beta^2v_d\mu v+4\beta^2v_d{}^2-2\beta\}f_0. \quad \text{(A2)}$$

Inserting the expression of Eq. (A2) into Eq. (A1) we can know that the first intergral at the right-hand side of Eq. (A1) vanishes.

From Eqs. (5), (11), and (12) $f_{1,3}{}^{r(1)}$ becomes

$$f_{1,3}{}^{r(1)}=-\frac{n\Gamma}{\omega}\left(\frac{1-\mu^2}{2v^3}\frac{\partial^2 f_1{}^{i(0)}}{\partial\mu^2}-\frac{\mu}{v^3}\frac{\partial f_1{}^{i(0)}}{\partial\mu}\right)$$
$$=\frac{n\Gamma}{m\omega^2}eE_1\{-4\beta^3v_d{}^2\mu^3+4\beta^2v_d{}^2\mu-(4\beta^3v_d{}^3\mu^2-8\beta^2v_d\mu^2+4\beta^2v_d-4\beta^3v_d{}^3)/v+(4\beta^2v_d{}^2\mu-2\beta\mu)/v^2\}f_0. \quad \text{(A3)}$$

By insertion of Eq. (A3) into Eq. (A1) the second integral in Eq. (A1) gives Eq. (20) together with Eqs. (21) and (22).